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CONTROL IN TECHNICAL SYSTEMS

A Hybrid Method for Integrating Separated Sensors for the Adaptive Identification of Stationary and Mobile Objects under Prior Uncertainty

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Abstract—A novel adaptive identification method for stationary and moving objects is developed within the combined application of the classical statistical approach based on probabilistic models and an unconventional approach oriented to the principles of continuity, multiplication, and clustering for a set of arbitrary spatially distributed sensors. The problem is solved under essential prior uncertainty associated, e.g., with the presence of weakly formalized abnormal measurement errors, faults and complete failures of individual sensors, or significant changes in the observation conditions of an identified object. Identification models, criterion, and algorithm robust to such uncertainty are presented. As a comparative analysis example, a triangulation system of sensors is used, and the effect achieved through adaptation is shown for this system. Practical recommendations are given.

Keywords: essential prior uncertainty, sensor, information and measurement system, identified object, integration, hybrid method, adaptation, abnormal measurement errors, continuity principle, multiplication principle, clustering principle, objective function, family of partial estimates, competing cluster, identification algorithm, posterior weights, resulting estimate

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1. INTRODUCTION

By now, many theoretical and practical advancements have been made for building various information and measurement systems (IMSs) consisting of spatially distributed sensors (homogeneous, inhomogeneous, stationary, and mobile), used for jointly identifying the state of stationary and moving objects. Such systems are widespread in different fields: medicine, fault diagnosis, radio astronomy, active and passive radar, electronic intelligence, field tests, security, etc.

Among the IMSs, there is an entire class of systems characterized by the following operating conditions: multistage data processing, real-time mode, essential uncertainty, and high risks (damage) from poor-quality assessments and decisions made on their basis. Uncertainty may be associated with unreliable knowledge of the distribution laws of measurement errors, skips in observations, the occurrence of equipment faults and complete failures, the presence of abnormal measurement errors (AMEs), etc.; for example, see [1–5]. Concerning possible risks, by assumption, it is important to consider not only the possible average damage but also the losses from a poor-quality solution in each particular case, i.e., when handling a fixed sample of measurements, particularly of a small size. Such samples are encountered at the stages of secondary and tertiary data processing, e.g., in active and passive radar systems (when forming the indirect measurements of bearings, time delays, slant ranges, radial velocities, etc.). The quality of such measurements determines the

efficiency of estimating the location and motion parameters of observed objects within the well-known coordinate measurement methods (triangulation, difference ranging, triangulation difference ranging, etc.).

Data processing algorithms under uncertainty are based on the classical probabilistic methods of maximum posterior probability density or maximum likelihood, usually in an extended version [1, 6–8]. In this version, all unknown uncertainty parameters are included in the expanded state vector, but in practice, the expansion procedure sharply increases computational cost and leads to the well-known effect of "smearing accuracy" and problems with the convergence and stability of the estimates. The above difficulties of classical methods are even further aggravated: within the multi-alternative approach [6], uncertainty is often eliminated by introducing a family of different hypotheses, each to be either rejected or confirmed. Analysis shows that for uncertainty conditions and strict requirements for IMSs, classical probabilistic methods are often of little use, but they can be effectively used at the mathematical modeling stage to study the potential capabilities of the IMSs being designed.

There exist other optimal and quasi-optimal estimation methods that can be used under uncertainty. They include the extended least squares (XLS) method [7], the minimum method of geometric and kinematic residuals [9], the method of invariants [10], various methods of linear and nonlinear filtering (e.g., Kalman) [1, 6, 7, 11], different adaptive methods with parameter tuning [12, 13], robust methods [6, 14, 15], randomized methods [16, 17], cluster methods [17, 18], and neural network methods [19], as well as numerous heuristics used in special-purpose IMSs (for example, see [20–25]). When imposing several constraints on observation conditions, these methods allow overcoming uncertainty factors (e.g., AMEs). However, under essential uncertainty, these methods encounter various unforeseen limitations and strict requirements for IMSs, which significantly narrow the scope of their application.

This is often due to the difficulty of considering the geometric factor, which depends on the spatial arrangement of the sensor family and the observed object. For example, triangulation-incorrect observation domains arise for goniometric systems, which are typical even in the case of good measurements. In addition, spatial estimation problems often involve pronounced nonlinearity, with the use of unbounded (e.g., some trigonometric functions, necessary for describing the relationship between the estimated and measured parameters), etc.

As a rule, the above methods guarantee convergence on average (either over an ensemble of realizations or on a single sufficiently long realization), not ensuring a high-quality estimate based on one small sample. In addition, the estimation procedures mainly use weight processing, in which weights are calculated based on a large volume of prior information. Under the specified uncertainty, such information may often be absent or turn out to be unreliable.

The general trend to build IMSs capable of withstanding different uncertainty factors is to integrate various estimation methods considering the limits of their applicability (including those mentioned above). Each method of a selected family should be "accuracy-orthogonal," in some sense, to other methods of this family and realize its potential capabilities for definite observation conditions. A hybrid method implementing the principle of integration should combine the advantages of both the classical statistical approach (ensuring optimality on average) and the unconventional approach (ensuring optimality in partial), covering to the maximum extent the specifics of an applied IMS and the uncertainty conditions.

For the unconventional approach, one can use the cluster-variant method [18], which has proven to be an effective tool for eliminating prior uncertainty, both on average and in partial. It involves the principle of multiplication of single estimates (considering a set of solution variations for the same problem with different sets of measurements based on an initial input observation), the clustering principle (dividing these estimates into several competing clusters), and an algorithm for

constructing the resulting estimate based on these clusters. This method suffers from the following drawbacks. First, only a triangulation system of goniometric (i.e., homogeneous) sensors is considered. Second, the identified object is supposed to be stationary (only its location is estimated). Third, the resulting estimate neglects the different accuracies of single estimates within one cluster. (This disagrees with the important principle of adaptability of the identification algorithm for a wider range of observation conditions.)

Below, we develop a generalized cluster-variant identification method (GCVIM) to completely remove all these disadvantages of the cluster-variant method [18]. The method should be based on principles understandable to experts in the field of construction and application of this class of IMSs in order to form guaranteed estimates (within the accepted constraints) of the location and, moreover, motion parameters of identified objects under essential uncertainty. The weights for measurement processing should take into account both prior and posterior information.

2. BASIC DEFINITIONS, MODELS, AND CONSTRAINTS. PROBLEM STATEMENT

By analogy with [18], to omit cumbersome notation and considerations, the method proposed is described mainly for the case of a single object. The method will be generalized to the case of several objects in Section 5.

In the Cartesian frame XYZ, we consider a system consisting of a set of sensors D_m, $m = \overline{1,M}$, where the position of each sensor is given by a vector $\boldsymbol{\rho}_m = [\rho_{xm}, \rho_{ym}, \rho_{zm}]^{\mathrm{T}}$. An object is characterized by the vector of estimated constant parameters $\boldsymbol{\lambda} = \left[\lambda_p, p = \overline{1,P}\right]^{\mathrm{T}}$, and all measured parameters are combined into the vector $\mathbf{s} = \left[s_j, j = \overline{1,J}\right]^{\mathrm{T}}$. For a moving object, the components of the vector $\boldsymbol{\lambda}$ represent the unknown spectral coefficients at the corresponding basis functions of a selected functional space. For a stationary object, $\boldsymbol{\lambda}$ includes the Cartesian coordinates of the object. Depending on the type of sensors used, the components of the vector $\mathbf{s}$ can be, e.g., the values of the following quantities: azimuth and elevation angle, slant range, radial velocity, as well as the level (amplitude or power), phase difference, or time delays of the received signal (including various derivatives of these quantities), related to different points in space and time instants.

The input observation is characterized by a vector

$$\mathbf{h} = \mathbf{s} + \boldsymbol{\Delta} + \boldsymbol{\xi}, \tag{1}$$

where $\boldsymbol{\Delta} = \left[\Delta_j, j = \overline{1,J}\right]^{\mathrm{T}}$ is an unknown measurement error and $\boldsymbol{\xi} = \left[\xi_j, j = \overline{1,J}\right]^{\mathrm{T}}$ is measurement noise with an unknown distribution law but zero mean and a given correlation matrix $\mathbf{K}_{\boldsymbol{\xi}}$.

To describe the error $\boldsymbol{\Delta}$, we use the following approach. Let $J^{\boldsymbol{\Delta}}$ correspond to the real number of those nonzero components of the vector $\boldsymbol{\Delta}$ that are AMEs and lead to unreliable identification results. Assume that $J^{\boldsymbol{\Delta}} \leqslant J^{\boldsymbol{\Delta}}_{\max}$, where $J^{\boldsymbol{\Delta}}_{\max} \in \{0, 1, \ldots\}$, and

$$J^{\mathbf{h}}_{\min} + J^{\boldsymbol{\Delta}}_{\max} \leqslant J, \tag{2}$$

where $J^{\mathbf{h}}_{\min}$ is the minimum number of components of the vector $\mathbf{h}$ sufficient for the high-quality identification of the object, given no AMEs.

Below, various sets of measurements will be formed on the basis of $\mathbf{h} : \mathbf{h}_i = \left[h_{ij}, j = \overline{1,J_i}\right]^{\mathrm{T}}$, where $h_{ij} \in \left\{h_j, j = \overline{1,J}\right\}$, $J_i \leqslant J$, $\mathbf{h}_i \neq \mathbf{h}_r$ $\forall i \neq r$, $i, r = \overline{1,I}$. For these sets, we introduce the constraint

$$J^{\mathbf{h}}_{\min} \leqslant J_i \leqslant J. \tag{3}$$

With this descriptive approach to the error $\mathbf{\Delta}$, some of its nonzero components can take arbitrary values (including anomalous), they can be located in the vector $\mathbf{\Delta}$ in the most arbitrary way, and there is no universal and satisfactory model to describe them. The only way to formalize the error $\mathbf{\Delta}$ is to introduce some quantitative constraints (like (2) and (3)), which correspond to the operation practice of a particular IMS.

The parameter vector $\boldsymbol{\lambda}$ is subject to the traditional constraints

$$\boldsymbol{\lambda} \in \Lambda \subset R^P, \tag{4}$$

where $\Lambda = \{\Lambda^p_{\min} \leqslant \lambda_p \leqslant \Lambda^p_{\max}\}$, $\Lambda^p_{\min} \in R^1$, and $\Lambda^p_{\max} \in R^1$.

By assumption, we know an operator $\mathbf{\Psi}$ such that

$$\mathbf{\Psi} : \mathbf{s} \to \boldsymbol{\lambda}, \tag{5}$$

i.e., $\boldsymbol{\lambda} = \mathbf{\Psi}(\mathbf{s})$; in addition, there exists an inverse operator

$$\mathbf{\Psi}^{-1} : \boldsymbol{\lambda} \to \mathbf{s} \tag{6}$$

for which $\mathbf{s} = \mathbf{\Psi}^{-1}(\boldsymbol{\lambda})$.

GCVIM involves three major principles: continuity, multiplication, and clustering. According to the first principle, the quality of identification continuously depends on the parameters of the input observation of a sufficient dimension. In particular, a sequential decrease in the parameter J (by excluding certain components of the vector $\mathbf{h}$) smoothly improves the accuracy of the resulting estimate. The second principle states that it is possible to form a set of partial estimates (both good and bad) and, due to conditions (2)–(4), the set of good estimates has sufficient cardinality. By the third principle, under conditions (2)– (4), all possible partial estimates can be clustered, and among the competing clusters there exists an optimal one, a basis for forming a reliable resulting estimate.

According to the multiplication principle, we associate the main vector $\mathbf{h}$ with a family of partial vectors $\left\{\mathbf{h}_i, i = \overline{1, I}\right\}$:

$$\mathbf{h}_i = \mathbf{s}_i + \mathbf{\Delta}_i + \boldsymbol{\xi}_i, \tag{7}$$

where $\mathbf{s}_i = \left[s_{ij}, j = \overline{1, J_i}\right]^{\mathrm{T}}$, $\mathbf{\Delta}_i = \left[\Delta_{ij}, j = \overline{1, J_i}\right]^{\mathrm{T}}$, and $\boldsymbol{\xi}_i = \left[\xi_{ij}, j = \overline{1, J_i}\right]^{\mathrm{T}}$.

The choice of I and $J_1, \ldots, J_I$ should agree with conditions (2) and (3); moreover, among $\left\{\mathbf{h}_i, i = \overline{1, I}\right\}$ there should exist vectors without AMEs.

Each vector $\mathbf{h}_i$ can be assigned a certain operator $\mathbf{\Psi}^*$ of classical optimal estimation (e.g., the maximum likelihood operator):

$$\mathbf{\Psi}^* : (\mathbf{h}_i, \mathbf{K}_{\boldsymbol{\xi}_i}) \to \boldsymbol{\lambda}^*_i, \tag{8}$$

where $\boldsymbol{\lambda}^*_i = \mathbf{\Psi}^*(\mathbf{h}_i, \mathbf{K}_{\boldsymbol{\xi}_i})$ is a partial estimate characterized by the correlation matrix $\mathbf{K}^*_i$ of estimation errors and $\mathbf{K}_{\boldsymbol{\xi}_i}$ is the correlation matrix of the noise $\boldsymbol{\xi}_i$.

The elements of the matrix $\mathbf{K}^*_i$ are calculated using well-known formulas (for example, see [1, 3, 4]); they are completely determined by the IMS type (triangulation, difference ranging, angle-power, etc.), its geometry and observation conditions, as well as the matrix $\mathbf{K}_{\boldsymbol{\xi}_i}$.

For each estimate $\boldsymbol{\lambda}^*_i$, the vector of secondary measurements $\left(\mathbf{s}^*_i = \left[s^*_{ij}, j = \overline{1, J_i}\right]^{\mathrm{T}}\right)$ is estimated using the known operator $\mathbf{\Theta}$ (which also depends on the type, geometry, and parameters of the IMS):

$$\mathbf{\Theta} : \boldsymbol{\lambda}^*_i \to \mathbf{s}^*_i, \tag{9}$$

where $\mathbf{s}^*_i = \mathbf{\Theta}\,(\boldsymbol{\lambda}^*_i)$.

Note that generally, the dimension J_i of the vector $\mathbf{s}_i^*$ does not coincide with that (J) of the vector $\mathbf{s}$. The vector $\mathbf{s}_i^*$ is calculated by considering the Cartesian coordinates of only those IMS sensors that are used to form the partial estimate vector $\mathbf{h}_i$.

Secondary measurements allow discarding bad partial estimates $\boldsymbol{\lambda}_i^*$ by checking the condition

$$\left|s_{ij}^* - h_{ij}\right| \leqslant \varepsilon_{ij} \quad \forall j = \overline{1, J_i}, \tag{10}$$

where ε_{ij} is the maximum value of the residual calculated considering the IMS type and the matrix $\mathbf{K}_i^*$.

If condition (10) fails or the estimate $\boldsymbol{\lambda}_i^*$ does not agree within the constraint (4), this partial estimate will be eliminated. Let $\boldsymbol{\lambda}_{[n]}^*$, $n = \overline{1, N}$ denote the partial estimates remaining after elimination, where $\boldsymbol{\lambda}_{[n]}^* \in \{\boldsymbol{\lambda}_1^*, \ldots, \boldsymbol{\lambda}_I^*\}$. Obviously, the set of such estimates is non-empty under conditions (2) and (3).

Definition 1. A scalar error Δ_{ij}, representing one component of the vector $\boldsymbol{\Delta}_i$, is said to be called abnormal if it violates condition (10). Otherwise, the error Δ_{ij} is said to be normal.

Definition 2. Any group of scalar errors (two or more) representing components of the vector $\boldsymbol{\Delta}_i$ is said to be anomalous (even if each of these components is itself normal) if it violates condition (10). Otherwise, the group is said to be normal.

Definition 3. The "clogging" coefficient of an original sample $\mathbf{h}$ is the value $k_{\mathbf{h}} = 100(k_{\boldsymbol{\Delta}}/J)[\%]$, where $k_{\boldsymbol{\Delta}}$ denotes the total number of abnormal scalar errors and scalar errors included in abnormal groups.

The generated set $\mathbf{X}^* = \{\boldsymbol{\lambda}_{[1]}^*, \ldots, \boldsymbol{\lambda}_{[N]}^*\}$ is divided into competing clusters K_q, $q = \overline{1, Q}$. (Any of the well-known clustering algorithms can be used, e.g., [18, 26, 27].) The estimates $\boldsymbol{\lambda}_{[1]}^*, \ldots, \boldsymbol{\lambda}_{[N]}^*$ are sequentially (step by step) combined into groups: first the closest, and then those with increasing distance to each other. At the first step, each estimate $\boldsymbol{\lambda}_{[n]}^*$ is treated as a separate cluster.

The clustering algorithm sets a mapping of the form $\Upsilon\colon \mathbf{X}^* \to \{\mathsf{K}_q, q = \overline{1, Q}\}$, and the following condition should hold:

$$\mathbf{X}^* = \bigcup_{q=1}^{Q} \mathsf{K}_q, \tag{11}$$

where $\mathsf{K}_k \cap \mathsf{K}_r = \varnothing$, $k, r \in \{1, \ldots, Q\}$, $k \neq r$, and $\mathsf{K}_q \neq \varnothing$ $\forall q = \overline{1, Q}$.

For each cluster, we will use the representation $\mathsf{K}_q = \left\{\boldsymbol{\lambda}_{[q,n]}^*, n = \overline{1, N_q}\right\}$, where $\boldsymbol{\lambda}_{[q,n]}^*$ is a partial estimate that falls into the cluster K_q and is assigned the number n. By analogy with (9), an estimate of the vector of secondary full-dimensional measurements $\mathbf{s}_{[q,n]}^* = \left[s_{j[q,n]}^*, j = \overline{1, J}\right]^{\mathrm{T}}$ is associated with each $\boldsymbol{\lambda}_{[q,n]}^*$:

$$\mathbf{s}_{[q,n]}^* = \boldsymbol{\Theta}\left(\boldsymbol{\lambda}_{[q,n]}^*\right). \tag{12}$$

By full-dimensionality we mean that the dimensions of the vectors $\mathbf{s}$ and $\mathbf{s}_{[q,n]}^*$ coincide.

The choice of the family of clusters $\{\mathsf{K}_1, \ldots, \mathsf{K}_Q\}$ was discussed in detail in [18]; in particular, a new adaptive clustering algorithm was proposed therein, including the case of unknown Q. This algorithm divides the entire family $\left\{\boldsymbol{\lambda}_{[1]}^*, \ldots, \boldsymbol{\lambda}_{[N]}^*\right\}$ into clusters autonomously (without operator intervention), is simple enough for computer implementation, and requires reasonable computational cost. According to the application results, this algorithm has high effectiveness and good asymptotic properties.

Based on (1)–(12), it is required to develop an adaptive GCVIM, in an optimal formulation, that has robustness to the error $\boldsymbol{\Delta}$ and constructs the resulting estimate $\boldsymbol{\lambda}^* = \left[\lambda_p^*, p = \overline{1,P}\right]^{\mathrm{T}}$ considering the different accuracies of partial estimates included in competing clusters. The method should include a justification of the objective function, criterion, and algorithm for selecting an optimal cluster and constructing an optimal resulting estimate.

3. JUSTIFICATION AND CONSTRUCTION OF THE OBJECTIVE FUNCTION. CRITERION AND IDENTIFICATION ALGORITHM

In the case where $\boldsymbol{\Delta} = \mathbf{0}$ and the only prior information available about the noise $\boldsymbol{\xi}$ is the correlation matrix $\mathbf{K}_{\boldsymbol{\xi}}$, preference is given to the traditional statistical method, which uses the weight matrix $\mathbf{W}_{\boldsymbol{\xi}} = \mathbf{K}_{\boldsymbol{\xi}}^{-1}$. (This ensures optimization on average.) Within GCVIM, under essential uncertainty, it is necessary to form a new objective function that would incorporate all the parameters figuring in the constraints (2)–(4) and characterizing competing clusters. This function should include weights that, unlike any optimal statistical method, are calculated a posteriori based on one fixed sample of measurements. (This ensures optimization in partial.)

To solve the identification problem, we use the following dimensionless normalized objective function:

$$F(\boldsymbol{\lambda}, q) = J^{-1} \sum_{j=1}^{J} \left\{ \left[\frac{s_j(\boldsymbol{\lambda}) - h_j}{\delta_j} \right]^2 w(h_j, q) \right\}, \quad 0 \leqslant F(\boldsymbol{\lambda}, q) \leqslant 1, \tag{13}$$

where $s_j(\boldsymbol{\lambda})$ is a known dependence of the measured parameter s_j on $\boldsymbol{\lambda}$; $w(h_j, q)$ is the posterior weight of the measurement h_j, $0 \leqslant w(h_j, q) \leqslant 1$; $\delta_j > 0$ is a coefficient limiting the value $|s_j(\boldsymbol{\lambda}) - h_j|$ for all viewed (model) values of $\boldsymbol{\lambda} \subset \Lambda$ and providing the corresponding normalization.

To find the weights, by analogy with (10), it is necessary to check the condition

$$\left| s_{j[q,n]}^* - h_j \right| \leqslant \varepsilon_{j[q,n]} \tag{14}$$

for the cluster K_q and any of partial estimates $\boldsymbol{\lambda}_{[q,n]}^*$ in this cluster.

The left-hand side in (14) characterizes the residual between the primary h_j and secondary $s_{j[q,n]}^*$ measurements for all possible values of j, q, and n. To consider the constraint on this residual, we use the indicator function

$$\varphi(p) = \begin{cases} 1 - p, & 0 \leqslant p \leqslant 1, \\ \varphi(p) = 0, & p > 1. \end{cases} \tag{15}$$

With $p = \left(s_{j[q,n]}^* - h_j\right)^2 / \varepsilon_{j[q,n]}^2$, the weights covering all the accepted constraints of the identification problem and the results of multiplying the partial estimates and their clustering, are determined as

$$w(h_j, q) = N_q^{-1} \sum_{n=1}^{N_q} \varphi \left(\frac{\left(s_{j[q,n]}^* - h_j\right)^2}{\varepsilon_{j[q,n]}^2} \right), \quad 0 \leqslant w(h_j, q) \leqslant 1, \tag{16}$$

where N_q^{-1} acts as a normalizing factor.

Formula (16) takes into account the number N_q of partial estimates in each cluster K_q, the thresholds $\varepsilon_{j[q,n]}$ of admissible errors in secondary measurements, and the presence of gross measurement errors (single and group) by introducing the indicator function $\varphi(p)$. The coefficient

$w(h_j, q)$ shows the specific contribution of the measurement h_j to the formation of all marks of the cluster K_q. It can be called the coefficient of matching between the measurement h_j and the elements of the cluster K_q. The higher value this coefficient takes, the more confidence there will be in the corresponding primary measurement h_j.

In view of (14)–(16), the desired objective function takes the form

$$F(\boldsymbol{\lambda}, q) = (JN_q)^{-1} \sum_{j=1}^{J} \sum_{n=1}^{N_q} \left[\frac{s_j(\boldsymbol{\lambda}) - h_j}{\delta_j} \right]^2 \varphi \left(\frac{\left(s^*_{j[q,n]} - h_j \right)^2}{\varepsilon^2_{j[q,n]}} \right). \tag{17}$$

The optimal identification criterion becomes

$$\boldsymbol{\lambda}^* = \arg\min_{\boldsymbol{\lambda}} F\left(\boldsymbol{\lambda}, q^*\right), \tag{18}$$

$$q^* = \arg\max_{q} w(\mathbf{h}, q) = \arg\max_{q} \left\{ J^{-1} \sum_{j=1}^{J} w(h_j, q) \right\}, \quad 0 \leqslant w(\mathbf{h}, q) \leqslant 1, \tag{19}$$

where $w(\mathbf{h}, q)$ is an integral dimensionless normalized weight.

Minimizing the decision function $F(\boldsymbol{\lambda}, q)$ with respect to the vector argument $\boldsymbol{\lambda}$ leads to the equation

$$\left[\frac{\partial F(\boldsymbol{\lambda}, q)}{\partial \boldsymbol{\lambda}} \right]^{\mathrm{T}} = \mathbf{0}. \tag{20}$$

By expanding the partial derivatives in (20), we obtain the system of scalar equations

$$\sum_{j=1}^{J} \left\{ \left(\frac{s_j(\boldsymbol{\lambda}) - h_j}{\delta_j} \right) w(h_j, q) \frac{\partial s_j(\boldsymbol{\lambda})}{\partial \lambda_p} \right\} = 0, \quad p = \overline{1, P}, \tag{21}$$

where partial derivatives are expanded given the known dependence $\mathbf{s} = \boldsymbol{\Psi}^{-1}(\boldsymbol{\lambda})$.

System (21) yields the partial estimates $\boldsymbol{\lambda}^*(q)$, $q = \overline{1, Q}$. As the resulting estimate $\boldsymbol{\lambda}^* \in \{\boldsymbol{\lambda}^*(1), \ldots, \boldsymbol{\lambda}^*(Q)\}$, we choose the partial estimate $\boldsymbol{\lambda}^*(q^*)$, $q^* \in \{1, \ldots, Q\}$, that satisfies, according to (19), the following criterion for a fixed $\mathbf{h}$:

$$w(\mathbf{h}, q^*) > w(\mathbf{h}, q) \quad \forall q = \overline{1, Q}, \quad q \neq q^*. \tag{22}$$

By choosing appropriate values of the parameters in conditions (2)–(4) and (10) and introducing appropriate ranges for the values of I, Q, and $\mathbf{K}_{\boldsymbol{\xi}}$, one can always ensure a unique solution of the identification problem with a required accuracy. With the correct planning of the measurement experiment (for example, see [7]), the resulting cluster K_{q^*} with the largest number of partial estimates formed by the reliable observations $\mathbf{h}_i$ is almost always detected. In turn, the parameters $\varepsilon_{j[q,n]}$ significantly affect the accuracy of identification.

The introduction of constraints (4) allows cutting the so-called incorrect identification domains, in which the uniqueness condition of the solution may be violated. For example, consider a triangulation IMS consisting of two direction finders; for such domains, the triangle formed by the base and two lines of sight has a too small angle at its apex (the object is located in it). In these cases, the lines of sight are almost parallel, and even minor bearing errors, not to mention AMEs, can cause significant coordinate measurement errors. In addition, there may appear competing clusters that are in the same direction, are separated by large distances, and have approximately the same number of elements. Such incorrectness is tackled either by introducing constraints (4) or

by using additional measurements from some external system that, together with the IMS under consideration, eliminates incorrect domains. This also applies to optimal experimental planning problems.

If we consider only one cluster, remove the accepted constraints, and let the weights $w(h_1, q), \ldots, w(h_J, q)$ be the reciprocal values of the variances of the primary measurement errors (distributed according to the Gaussian law), the solution (18)–(21) will correspond to the well-known maximum likelihood estimate for the case of no AMEs.

In the classical statistical formulation, the identification problem is solved based on all measurements (regardless of whether they are good or bad), which undergo joint optimal processing considering pre-assigned prior weights. The novel hybrid method also operates all measurements but takes into account both prior information (in the form of the weight matrices $\mathbf{W}_{\boldsymbol{\xi}_i} = \mathbf{K}_{\boldsymbol{\xi}_i}^{-1}$ used to construct partial estimates) and posterior information (in the form of the weights $w(h_1, q), \ldots, w(h_J, q)$ and $w(\mathbf{h}, q)$), which is required to find the optimal cluster and construct the resulting estimate.

The algorithm for solving the identification problem in the cluster optimal formulation includes the following steps:

1) The partial estimates are divided into clusters K_q, $q = \overline{1, Q}$ (see clustering recommendations in [18]).

2) For each cluster K_q, the number of marks (N_q) in it, as well as the partial $w(h_1, q), \ldots, w(h_J, q)$ and integral $w(\mathbf{h}, q)$ weights, are calculated.

3) The number q^* of the optimal cluster K_{q^*} is found using the criterion (19).

4) For the cluster K_{q^*}, the system of equations (21) is solved, with the initial condition in the form of a partial estimate taken as the center of this cluster. Finally, in view of (21), we obtain the resulting estimate $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^*(q^*) = \left[\lambda_p(q^*), p = \overline{1, P}\right]^{\mathrm{T}}$ of the parameter vector $\boldsymbol{\lambda}^*$ of the identified object.

Remark 1. Based on the linear approximation of the residuals $s_j(\boldsymbol{\lambda}) - h_j$, the well-known approximate approach to constructing the maximum likelihood estimate yields the corresponding system of linear algebraic equations instead of (21).

The implementation of the proposed algorithm (with the preparation of initial data) requires the computational cost $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$, where Γ_1 is the cost of constructing a family of partial estimates, Γ_2 is the cost of clustering these estimates and selecting the resulting cluster, and Γ_3 is the cost of obtaining the resulting estimate. The implementation of these operations in special computing environments causes no particular difficulties and can be done in real time.

Formulas (13)–(22) form the mathematical basis of GCVIM.

4. APPLICATION OF THE NOVEL METHOD TO A TRIANGULATION IMS

Consider a triangulation IMS with two-channel direction finders used as sensors ($\mathrm{D}_m, m = \overline{1, M}$), to measure the azimuth α_m and elevation angle β_m of a stationary emitting object. In this case, we have $P = 3$, $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^{\mathrm{T}} = [x, y, z]^{\mathrm{T}}$, $J = 2M$, $\mathbf{s} = \left[\alpha_m, \beta_m, m = \overline{1, M}\right]^{\mathrm{T}}$, $\mathbf{h} = \left[h_j, j = \overline{1, J}\right]^{\mathrm{T}} = \left[h_m^{\alpha}, h_m^{\beta}, m = \overline{1, M}\right]^{\mathrm{T}}$, and $\delta_j = \pi$.

As $\mathbf{s} = \boldsymbol{Psi}^{-1}(\boldsymbol{\lambda})$ the well-known formulas are applied:

$$\begin{cases} \alpha_m(\boldsymbol{\lambda}) = \arccos\left\{(x - x_m)\left[(x - x_m)^2 + (y - y_m)^2\right]^{-1/2}\right\}, \\ \beta_m(\boldsymbol{\lambda}) = \arcsin\left\{(z - z_m)\left[(x - x_m)^2 + (y - y_m)^2 + (z - z_m)^2\right]^{-1/2}\right\}. \end{cases} \tag{23}$$

The system of equations (21) is formed based on (23).

Condition (14) turns into

$$\begin{cases} \left|\alpha^*_{m[q,n]} - h^\alpha_m\right| \leqslant \varepsilon^\alpha_{m[q,n]}, \\ \left|\beta^*_{m[q,n]} - h^\beta_m\right| \leqslant \varepsilon^\beta_{m[q,n]}, \end{cases} \tag{24}$$

and the objective function becomes

$$F(\boldsymbol{\lambda}, q) = \sum_{m=1}^{M} \left[\left(\frac{\alpha_m(\boldsymbol{\lambda}) - h^\alpha_m}{\pi} \right)^2 w(h^\alpha_m, q) + \left(\frac{\beta_m(\boldsymbol{\lambda}) - h^\beta_m}{\pi} \right)^2 w\left(h^\beta_m, q\right) \right], \tag{25}$$

$$\begin{cases} w\left(h^\alpha_m, q\right) = N_q^{-1} \sum\limits_{n=1}^{N_q} \varphi\left(\dfrac{\left(\alpha^*_{m[q,n]} - h^\alpha_m\right)^2}{\left(\varepsilon^\alpha_{m[q,n]}\right)^2} \right), \\ w\left(h^\beta_m, q\right) = N_q^{-1} \sum\limits_{n=1}^{N_q} \varphi\left(\dfrac{\left(\beta^*_{m[q,n]} - h^\beta_m\right)^2}{\left(\varepsilon^\beta_{m[q,n]}\right)^2} \right). \end{cases} \tag{26}$$

The integral weight is calculated as follows:

$$w(\mathbf{h}, q) = M^{-1} \sum_{m=1}^{M} \left[w(h^\alpha_m, q) + w(h^\beta_m, q) \right]. \tag{27}$$

As an example, we take the planar problem by letting $\rho_{zm} = 0$ and $\lambda_3 = z = 0$, i.e., $\beta_m = 0 \ \forall m = \overline{1, M}$. Assume that the matrix of primary azimuthal errors is diagonal: $\mathbf{K}_{\boldsymbol{\xi}} = \operatorname{diag}[\sigma^2_{\alpha 1}, \ldots, \sigma^2_{\alpha M}]$. In this case, using the known operator $\boldsymbol{\Psi}^*$, we can form a family of partial estimates $\boldsymbol{\lambda}^*_i = [x^*_i, y^*_i]^{\mathrm{T}}$, where

$$\begin{cases} x^*_i = (B_i E_i - C_i D_i)(A_i C_i - B_i^2)^{-1}, \\ y^*_i = (B_i D_i - A_i E_i)(A_i C_i - B_i^2)^{-1}. \end{cases}$$

Here, $A_i = \sum_{m=1}^{M_i} \cos^2 \alpha_{mi}/\gamma_{mi}$, $B_i = 2^{-1} \sum_{m=1}^{M_i} \sin 2\alpha_{mi}/\gamma_{mi}$, $C_i = \sum_{m=1}^{M_i} \sin^2 \alpha_{mi}/\gamma_{mi}$, $D_i = \sum_{m=1}^{M_i} \varphi_{mi}/\gamma_{mi}$, $E_i = \sum_{m=1}^{M_i} \mu_{mi}/\gamma_{mi}$, $\gamma_{mi} = \widetilde{R}^2_{mi}\sigma^2_{\alpha mi}$, $\varphi_{mi} = \rho_{xmi} \cos^2 \alpha_{mi} - 2^{-1}\rho_{ymi} \sin 2\alpha_{mi}$, $\mu_{mi} = 2^{-1}\rho_{xmi} \sin 2\alpha_{mi} - \rho_{ymi} \sin^2 \alpha_{mi}$, and $\widetilde{R}_{mi}$ is the approximate range from the object to the sensor D_{mi}, $m \in \{1, \ldots, M_i\}$.

For the triangulation IMS, we construct estimation error ellipses and, in view of the partial estimates and the geometry of the triangulation IMS, use these ellipses to calculate the constants $\varepsilon^\alpha_{m[q,n]}$ for each cluster and partial estimates in it.

Formulas (25)–(27) form the basis of the one-stage GCVIM for the triangulation IMS. Unlike the well-known cluster-variant triangulation estimation method [18], when forming posterior weights, the novel method considers the fact that the right-hand sides of inequalities (24) depend both on the competing cluster number q and on the partial estimate number n in this cluster.

It is also possible to use the two-stage GCVIM for the triangulation IMS in two versions. The first version is based on the possibility to initially implement the two-stage GCVIM only on azimuthal channels (the first stage) to form a family of planar partial estimates and clusters and then determine the estimates x^* and y^* of the two Cartesian coordinates of the object (x and y, respectively).

A particular line of the object's position corresponds to the point (x^*, y^*) in the three-dimensional space. Combining this line and all possible position cones corresponding to the elevation angles, one constructs the missing estimate z^* of the spatial coordinate z (the second stage).

The second version is to find, in relation only to azimuth channels, unreliable azimuth channels using the matching coefficients $w\left(h_m^{\alpha}, q\right)$. The measurements of such channels are excluded from further processing. Then, on the remaining (priority) azimuth channels and all elevation channels, GCVIM is implemented in full. In this case, the number of partial estimates analyzed increases (compared to the first version), which improves the quality of triangulation estimation under uncertainty.

Obviously, the two-stage GCVIM for the two versions above does not exhaust the potential capabilities of the optimal GCVIM under uncertainty but is less costly from a computational viewpoint. This is primarily due to a significant reduction in the number of partial triangulation estimates and competing clusters.

5. GENERALIZATION OF THE METHOD TO THE CASE OF MANY OBJECTS

The extension of GCVIM to this case depends on the purpose, design principles, and organization of the computing process in a particular IMS. We mention two basic versions of GCVIM implementation in the case of many objects.

Version 1. A separate stage is to divide measurements into classes based on their belonging to a given object (the measurement identification problem). Such a truncated problem formulation is quite common in practice under the corresponding decomposition of the computational process. In this case, GCVIM is applied to each class identified. For this, it is necessary to form the measurement vectors $\mathbf{h}_{(l)} = \left[h_{(l)j}, j = \overline{1, J_{(l)}}\right]^{\mathrm{T}}$, where $l \in \overline{1, L}$ denotes the object (class) number. For the lth class, by analogy with Section 3, the elements $\mathsf{K}_{(l)q}$, $M_{(l)q}$, $w_{(l)}(h_j, q)$, $w_{(l)}(\mathbf{h}, q)$, and $q^*_{(l)}$ are formed, and then the resulting estimates $\boldsymbol{\lambda}^*_{(l)} = \left[\lambda^*_{(l)j}, j = \overline{1, J}\right]^{\mathrm{T}}$, $l = \overline{1, L}$, are constructed on their basis. Version 1, the most efficient from a computational viewpoint, allows organizing L parallel processing channels for measurements, but it does not exhaust the potential joint processing capabilities for available measurements.

Note 1. If incorrect decisions are made when solving the identification problem, then the false measurements falling into a wrong class can be treated as measurements with gross errors. GCVIM allows one to effectively struggle with such errors regardless of their nature.

Note 2. It is possible to solve the identification problem by engaging ancillary measurements, which often "load" basic measurements. (For example, in radar and electronic intelligence, these may include the carrier frequency, pulse repetition period, pulse duration, the type of intra-pulse modulation, etc.) The joint use of basic and ancillary measurements is often necessary for deeper identification of objects.

Version 2. In this case, GCVIM is applied to all measurements at once, forming clusters K_q, $q = \overline{1, Q}$. Initially, it is necessary to determine the numbers $q^*_{(l)} \in \{1, 2, \ldots, Q\}$ of priority (optimal) clusters $\mathsf{K}_{q^*_{(l)}}$, which is achieved by introducing the criterion $w_{(l)}\left(\mathbf{h}, q^*_{(l)}\right) \geqslant \gamma$, where γ is a given threshold for object recognition, $\gamma > 0$. Next, for all these clusters, the resulting estimates are constructed: $\boldsymbol{\lambda}^*_{(l)} = \left[\lambda^*_{(l)j}, j = \overline{1, J}\right]^{\mathrm{T}}$, $l = \overline{1, L}$.

Version 2 is quite costly from a computational viewpoint, as it yields a large number Q of clusters. The advantage of this version is that the identification problem for the case of many objects realizes the potential capabilities of GCVIM under uncertainty.

6. COMPARATIVE ANALYSIS

Consider a triangulation IMS in which the sensors D_m are located on a circle: $\boldsymbol{\rho}_m = [\rho_{xm}, \rho_{ym}, \rho_{zm}]^{\mathrm{T}}$, $\rho_{xm} = 10^4 \cos[2\pi(m-1)/M]$, $\rho_{ym} = 10^4 \sin[2\pi(m-1)/M]$, $\rho_{zm} = 0$, and $M = 5$, $m = \overline{1,5}$. For a stationary object, we investigated the set of its possible positions, with each position described by an individual vector of the form $\boldsymbol{\lambda}_k = [x_k, y_k, z_k]^{\mathrm{T}}$, $x_k = 5 \times 10^4 \cos(2\pi k/K)$, $y_k = 5 \times 10^4 \sin(2\pi k/K)$, $z_k = 3 \times 10^3$, $k = \overline{1,K}$, where $K = 180$. From this point onwards, the coordinates of the object and sensors are specified in meters whereas the azimuth, elevation angle, and bearing errors in radians. For each k, it was assumed that the measurement noises of different equally accurate sensors are independent and obey the Gaussian distribution with zero mean, and the correlation matrix of the sensor D_m has the diagonal form $\mathbf{K}_m = \mathrm{diag}[\sigma_\alpha^2, \sigma_\beta^2]$, where $\sigma_\alpha = \pi/360$ and $\sigma_\beta/360$. Errors were formed by a random number generator. The numbers of unreliable channels (no more than half of all azimuth channels and no more than half of all elevation channels) containing AMEs were selected randomly. In the channels, the resulting measurement error was formed as the sum of the AME and the admissible random error, with the possible ranges $(3\sigma_\alpha, \pi/6)$ for the azimuth channel and $(3\sigma_\beta, \pi/6)$ for the elevation channel.

For comparative analysis, four algorithms corresponding to the well-known cluster-variant method (CVM, see [28]) and GCVIM (the novel method) were considered: A_1 is the CVM algorithm with a predetermined number Q of clusters; A_2 and A_3 are the one- and two-stage CVM algorithms, respectively, with an optimally chosen number Q of clusters for a given initial condition Q_0; A_4 is the GCVIM algorithm. The algorithms were compared in terms of accuracy and computational efficiency.

The estimation procedure was carried out for each k with subsequent averaging over 100 experiments. In Algorithm A_1, $Q = 7$; in Algorithms A_2 and A_3, $Q_0 = 2$ was set as the initial condition. Two numerical characteristics were used to compare the algorithms: $S(\mathrm{A}_i)$, the integral accuracy characteristic expressed in meters, and $T(\mathrm{A}_i)$, the computational efficiency characteristic expressed in seconds.

For the integral accuracy characteristic, we have

$$S(\mathrm{A}_i) = \sum_{k=1}^{180} S_k(\mathrm{A}_i) = \frac{2\pi}{180} \sum_{k=1}^{180} \tau_k(\mathrm{A}_i),$$

where $\tau_k(\mathrm{A}_i) = \left\| \overline{\boldsymbol{\lambda}}_k^*(\mathrm{A}_i) - \boldsymbol{\lambda}_k \right\|_2$ is the partial residual corresponding to the kth position of the object,

$$\overline{\boldsymbol{\lambda}}_k^*(\mathrm{A}_i) = \sum_{p=1}^{100} \boldsymbol{\lambda}_{kp}^*(\mathrm{A}_i)/100$$

is the average estimate of the vector $\boldsymbol{\lambda}_k$ (over 100 experiments), and $\boldsymbol{\lambda}_{kp}^*(\mathrm{A}_i)$ is the partial estimate for the kth position of the object in the pth experiment.

Algorithms A_i were compared in terms of the relative integral accuracy characteristic

$$\delta S(\mathrm{A}_i) = 100 S(\mathrm{A}_i) S^{-1}(\mathrm{A}_1)[\%]$$

and the relative computational efficiency

$$\delta T(\mathrm{A}_i) = 100 T(\mathrm{A}_i) T^{-1}(\mathrm{A}_4)[\%],$$

as A_1 is less accurate and A_4 requires more time compared to the other algorithms. The simulation results are presented in the table below.

Table

Algorithms (A_i)	A_1	A_2	A_3	A_4
$\delta S(\mathrm{A}_i)$, %	100	50	67	39
$\delta T(\mathrm{A}_i)$, %	81	92	14	100

Clearly, in the abnormal operation conditions of the triangulation IMS, the modified algorithm A_4, which considers the posterior change in weights depending on the object's position, the competing cluster number, and the position of each partial estimate in this cluster, is significantly better (in terms of accuracy) than the alternative algorithms. According to the simulation results, by analogy with [18], under the above abnormal observation conditions, the maximum likelihood method is not applicable in either a simple or extended version. For instance, the error of coordinate measurement based on the IMS under consideration reaches 100% in several directions.

7. CONCLUSIONS

The GCVIM proposed in this paper allows extending the idea of cluster-variant estimation from the simplest problem of determining the location of a stationary object in a triangulation IMS to a more complex one of identifying the parameter vector of a moving object in an arbitrary-type IMS with homogeneous and even heterogeneous measurements related to arbitrary points in space and time. The method produces a resulting estimate of the parameter vector that is robust to the effect of gross measurement errors (e.g., abnormal single and group ones) under essential prior uncertainty. GCVIM, both independently and in combination with traditional statistical approaches, can be effectively used to improve the existing and develop promising IMSs. In the case of limited computing resources and/or a large number of sensors and a high dimension of the observation vector, the two-stage version of GCVIM implementation can significantly increase the efficiency of identification.

Modern progress in the field of parallel computing (especially based on new operation principles [29, 30]) gives hope that the principle of multiplying partial estimates and their clustering in the optimal variant will not become an obstacle to the practical implementation of GCVIM in promising real-time IMSs. However, such an approach is feasible not for all existing IMSs: on large samples, it may require too many parallel data processing channels and significant computational cost.

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